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HEATING OF A COMPOSITE OF PERIODIC STRUCTURE IN A HIGH-FREQUENCY ELECTROMAGNETIC FIELD

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Composites based on glass, carbon, or boron fibers with epoxy resin as a binder are widely used in aeronautical, space, and rocket facilities, as well as in machine building [1, 2]. The properties of these composites can be improved with simultaneous simplification of production processes by using the energy of an electromagnetic field of super-high frequency (SHF) in the drying and solidification stages [2]. Using SHF-heating, i.e., volume, distant, controlled energy supply, one can decrease by severalfold the duration of drying and polymerization. However, volume heating with local energy release that depends on the electrophysical properties of the components can be accompanied by local superheating with undesired consequences for the quality of the products. Optimal heating regimes in an SHF field can be chosen by calculation of the thermal fields appearing in the composite under such heating, since direct experimental determination of thermal fields is hardly possible.

In this work, we present an analytical solution of the problem for a unidirectional composite with regular arrangement of fibers. This structure is the simplest from a technological viewpoint and is used as the model object.

1. We consider the heating of a composite in a quasi-stationary electromagnetic field (a high-frequency field with a slowly changing amplitude) [3]:

$$\mathbf{E}_*(\mathbf{x},t) = \mathbf{E}(\mathbf{x})\chi(t)\exp(-i\omega t), \tag{1.1}$$

where $|\partial\chi/\partial t| \ll \omega\chi$ and $|\partial^2\chi/\partial t^2| \ll \omega^2\chi$. The composite itself is simulated by an ideal periodic medium that is a combination of periodicity cells αY with a characteristic size α that is small compared with the body dimensions and with the length of electromagnetic waves that heat the composite.

The temperature field under these conditions is determined from the heat-conduction equation

$$\rho c\frac{\partial T}{\partial t} = \frac{\partial}{\partial x_i}\Big(\lambda\frac{\partial T}{\partial x_i}\Big) + D. \tag{1.2}$$

Here the density ρ, the specific heat c, the heat conductivity λ, and the specific capacity of the heat sources averaged over the vibration period $D = (1/2)\chi^2(t)\omega\varepsilon_*'' E_k\overline{E}_k$ are rapidly oscillating functions of the $\mathbf{x}$ coordinate (ε_*'' is the imaginary part of the complex dielectric permeability ε_* which takes into account both the conductivity and polarization relaxation of the medium). In (1.2) and below, summation is performed over the repeated subscripts i, j, $k = 1, 2, 3$.

In the case of ideal contact, the following conditions should be satisfied at the interface Γ of the composite:

$$[T] = 0, \quad \Big[\lambda\frac{\partial T}{\partial x_i}\Big]\nu_i = 0. \tag{1.3}$$

Here [ ] is the jump of the quantity and $\boldsymbol{\nu}$ is a unit normal to Γ directed toward the jump.

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2. We construct a solution of problem (1.1)–(1.3) as a series in small parameter ν using the averaging technique [4, 5]:

$$T(\mathbf{x},t) = T_0(\mathbf{x},\mathbf{y},t) + \alpha T_1(\mathbf{x},\mathbf{y},t) + \alpha^2 T_2(\mathbf{x},\mathbf{y},t) + \dots \; . \tag{2.1}$$

Here, along with the global space variables $\mathbf{x} = \{x_1, x_2, x_3\}$, we introduce the local variables $\mathbf{y} = \mathbf{x}/\alpha$. The functions T_i are periodic in the variable $\mathbf{y}$ with a periodicity cell Y. For brevity, we introduce the notation for the operator

$$L_{\xi\eta}(T) = \frac{\partial}{\partial \xi_i}\Big(\lambda \frac{\partial T}{\partial \eta_i}\Big).$$

Confining our consideration to macrohomogeneous media, we assume that ρ, c, λ and ε_* are Y-periodic functions only of the variable $\mathbf{y}$.

The specific capacity of the heat source D is determined by solution of the appropriate electrodynamic problem [6]:

$$\mathbf{E}(\mathbf{x}) = \mathbf{E}^{(0)}(\mathbf{x},\mathbf{y}) + \alpha \mathbf{E}^{(1)}(\mathbf{x},\mathbf{y}) + \dots \, ,$$

whence it follows that

$$\begin{gathered} D(\mathbf{x},\mathbf{y}) = D^{(0)}(\mathbf{x},\mathbf{y}) + \alpha D^{(1)}(\mathbf{x},\mathbf{y}) + \dots \\ [D^{(0)} = (1/2)\chi^2 \omega \varepsilon_*'' E_i^{(0)} \overline{E}_i^{(0)}, \quad E_i^{(0)}(\mathbf{x},\mathbf{y}) = (\delta_{ij} + \Phi_{j|i}(\mathbf{y}))\langle E_j^{(0)}\rangle(\mathbf{x})]. \end{gathered} \tag{2.2}$$

Here and below, differentiation with respect to y_i is denoted by a vertical bar, $(|i)$, and differentiation with respect to x_i is denoted by a comma, $(,i)$.

The functions $\Phi_j(\mathbf{y})$ are Y-periodic solutions of the local electrodynamic problem:

$$(\varepsilon_* \Phi_{j|i})_{|i} + \varepsilon_{*|j} = 0, \quad [\Phi_i]\big|_\Gamma = 0, \quad [\varepsilon^*(\delta_{ij} + \Phi_{j|i})]\big|_\Gamma \nu_i = 0. \tag{2.3}$$

Angle brackets $\langle\ \rangle$ denote an average value over the cell volume $|Y|$, for example,

$$\langle D^{(0)}\rangle(\mathbf{x}) = \frac{1}{|Y|}\int\limits_Y D^{(0)}(\mathbf{x},\mathbf{y})\,d\mathbf{y} = \frac{1}{2}\chi^2 \omega \langle \Delta_{ij}\rangle \langle E_i^{(0)}\rangle(\mathbf{x}) \langle E_j^{(0)}\rangle(\mathbf{x}) \tag{2.4}$$

$$[\Delta_{ij}(\mathbf{y}) = \varepsilon_*''(\mathbf{y})(\delta_{ij} + \Phi_{i|j})(\delta_{ij} + \Phi_{j|i})].$$

Substituting (2.1) and (2.2) in Eq. (1.2) and equating the coefficients of equal exponents α^{-2}, α^{-1}, and α^0, respectively, we obtain the equations

$$L_{yy} T_0 = 0; \tag{2.5}$$

$$L_{yy} T_1 = -(L_{yx} T_0 + L_{xy} T_0); \tag{2.6}$$

$$L_{yy} T_2 = \rho c \frac{\partial T_0}{\partial t} - D^{(0)} - (L_{yx} T_1 + L_{xy} T_1 + L_{xx} T_0). \tag{2.7}$$

It follows from (2.5) that T_0 is independent of the local variable $\mathbf{y}$:

$$T_0 = T_0(\mathbf{x},t). \tag{2.8}$$

With allowance for (2.8), Eq. (2.6) becomes

$$L_{yy} T_1 = -\lambda_{|i} T_{0,i}. \tag{2.9}$$

The Y-periodic solution of (2.9) is of the form

$$T_1(\mathbf{x},\mathbf{y}) = \Phi_j(\mathbf{y}) T_{0,j}(\mathbf{x}), \tag{2.10}$$

where $\Phi_j(\mathbf{y})$ ($j = 1, 2, 3$) is a periodic solution of the equations

$$(\lambda \Phi_{j|i})_{|i} + \lambda_{|j} = 0, \tag{2.11}$$

which differ from the electrodynamic equations (2.3) only by the substitution of $\lambda(\mathbf{y})$ for $\varepsilon_*(\mathbf{y})$. In view of (2.8) and (2.10), Eq. (2.7) becomes

$$L_{yy}T_2 = \rho c\,\frac{\partial T_0}{\partial t} - D^{(0)} - T_{ij}(\mathbf{y})T_{0,ij} \quad [T_{ij}(\mathbf{y}) = \lambda_{|i}\Phi_j + 2\lambda\Phi_{j|i} + \lambda\delta_{ij}]. \tag{2.12}$$

Averaging (2.12) over the cell volume and taking into account the Y-periodicity of $\Phi_j(\mathbf{y})$, we obtain the heat-conduction equation for an equivalent homogeneous medium:

$$\langle\rho c\rangle\frac{\partial T_0}{\partial t} = \widehat{\lambda}_{ij}T_{0,ij} + \langle D^{(0)}\rangle, \tag{2.13}$$

where $\widehat{\lambda}_{ij} = \langle\lambda(\delta_{ij} + \Phi_{j,i})\rangle$ is the effective heat conductivity of the composite.

Excluding the quantity $\partial T_0/\partial t$ from (2.12) and (2.13) and taking into account expression (2.4), we obtain

$$L_{yy}T_2 = \Big(\frac{\rho c}{\langle\rho c\rangle}\widehat{\lambda}_{ij} - T_{ij}\Big)T_{0,ij} + \frac{1}{2}\chi^2\omega N_{ij}\langle E_i^{(0)}\rangle\langle\overline{E}_j^{(0)}\rangle$$
$$[N_{ij} = N_{ij}(\mathbf{y}) = \frac{\rho c}{\langle\rho c\rangle}\langle\Delta_{ij}\rangle - \Delta_{ij}]. \tag{2.14}$$

The solution of Eq. (2.14) is representable as

$$T_2 = T_2(\mathbf{x},\mathbf{y}) = \Phi_{ij}T_{0,ij} + \frac{1}{2}\chi^2\omega\nu_{ij}\langle E_i^{(0)}\rangle\langle\overline{E}_j^{(0)}\rangle, \tag{2.15}$$

where the Y-periodic functions $\Phi_{ij} = \Phi_{ij}(\mathbf{y})$ and $\nu_{ij} = \nu_{ij}(\mathbf{y})$ are solutions of the equations

$$L_{yy}\Phi_{ij} = \frac{\rho c}{\langle\rho c\rangle}\widehat{\lambda}_{ij} - T_{ij}; \tag{2.16}$$

$$L_{yy}\nu_{ij} = N_{ij}. \tag{2.17}$$

It is not hard to verify that the condition of the theorem of existence and uniqueness of solutions of local problems [4], which reduces to the equality to zero of the mean value of the right-hand side, is satisfied for Eqs. (2.3), (2.11), (2.16), and (2.17).

Summing up, we find that the temperature distribution in the composite is of the form

$$T(\mathbf{x},\mathbf{y},t) = T_0(\mathbf{x},t) + \alpha\Phi_i(\mathbf{y})T_{0,i}(\mathbf{x},t)$$
$$+\alpha^2\Big[\Phi_{ij}(\mathbf{y})T_{0,ij}(\mathbf{x},t) + \frac{1}{2}\chi^2(t)\omega\nu_{ij}(\mathbf{y})\langle E_i^{(0)}\rangle(\mathbf{x})\langle\overline{E}_j^{(0)}\rangle(\mathbf{x})\Big] + \dots\,. \tag{2.18}$$

The first term on the right-hand side of (2.18) is the temperature $T_0(\mathbf{x},t) = \langle T(\mathbf{x},\mathbf{y},t)\rangle$ averaged over the structural-cell volume, or, in other words, the temperature field in the equivalent homogeneous material with effective characteristics $\widehat{\lambda}_{ij}$. The subsequent terms are local corrections to $T_0(\mathbf{x},t)$ of different orders of smallness, each having a zeroth mean value over the volume of the periodicity cell Y. It should be noted that in macrononuniform temperature fields, in which $T_{0,i} \neq 0$, the correction is on the order of α and is determined by the effective characteristics of the material $\widehat{\lambda}_{ij}$, $\langle\rho c\rangle$, and $\langle\Delta_{ij}\rangle$, while in macrouniform temperature fields, in which $T_{0,i} = T_{0,ij} = 0$, the correction is on the order of α^2 and depends on the distribution of sources over the cell:

$$T = T(\mathbf{y},t) = T_0(t) + \frac{1}{2}\alpha^2\chi^2(t)\omega\nu_{ij}(\mathbf{y})\langle E_i^{(0)}\rangle\langle\overline{E}_j^{(0)}\rangle + \dots\,. \tag{2.19}$$

The conditions at the interface Γ of the composite are obtained from (1.3) and (2.18):

$$[\Phi_i] = 0, \quad [\lambda(\delta_{ij} + \Phi_{j|i})]\nu_i = 0; \tag{2.20}$$

$$[\Phi_{ij}] = 0, \quad [\lambda(\Phi_k\delta_{ij} + \Phi_{jk|i})]\nu_i = 0; \tag{2.21}$$

$$[\nu_{ij}] = 0, \quad [\lambda\nu_{jk|i}]\nu_i = 0. \tag{2.22}$$

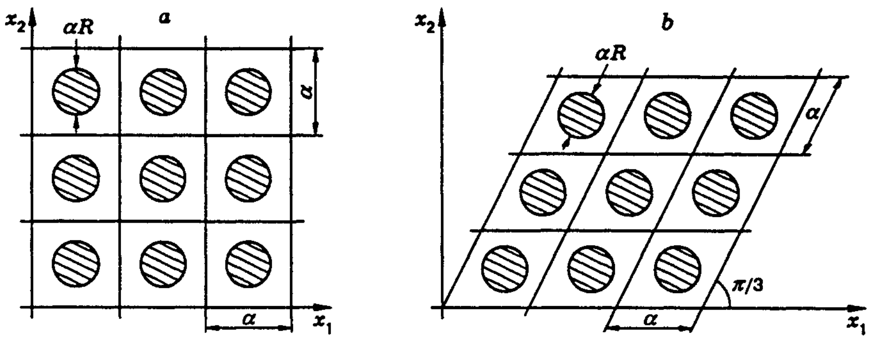

Fig. 1

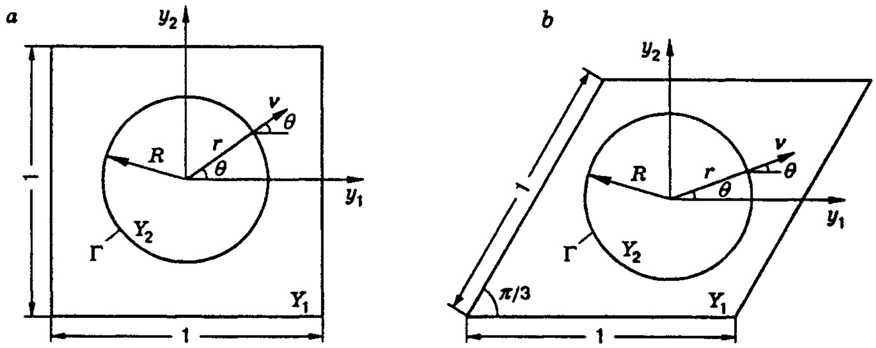

Fig. 2

3. We consider an idealized unidirectional composite whose fibers have a circular cross section with radius αR; the axes of the fibers pass through the centers of the periodicity cells Y perpendicular to the plane x_1, x_2. We confine ourselves to two cases of regular arrangement of fibers with either a regular square lattice (Fig. 1a) or a regular triangular lattice (Fig. 1b) in the cross section. The periodicity cells in the local system of the y_1, y_2 coordinates are a singular square or a rhomb with a unit side and an acute angle of 60° (Fig. 2a and 2b).

In what follows, we assume that the matrix Y_1 and the fiber Y_2 are homogeneous, and heat release occurs only in the fibers, i.e., $\lambda(\mathrm{y}) = \lambda_1 = \mathrm{const}$, $\varepsilon''_*(\mathrm{y}) = 0$ ($\mathrm{y} \in Y_1$), $\lambda(\mathrm{y}) = \lambda_2 = \mathrm{const}$, and $\varepsilon''_*(\mathrm{y}) = \varepsilon''_2 = \mathrm{const}$ ($\mathrm{y} \in Y_2$).

Because of the complete analogy of the local electrodynamic problems (2.3) and the heat-conduction problems (2.11) and (2.20), the solutions are described by the same harmonic functions $\Phi_1(y_1, y_2)$ and $\Phi_2(y_1, y_2)$ constructed in [6]. For example, for the fiber Y_2,

$$\Phi_1^{(2)} = R\sum_{k=0}^{\infty} A_{2k+1}^{(2)}(r/R)^{2k+1}\cos(2k+1)\theta, \quad \Phi_2^{(2)} = R\sum_{k=0}^{\infty}(-1)^k A_{2k+1}^{(2)}(r/R)^{2k+1}\sin(2k+1)\theta, \tag{3.1}$$

where $A_{2k+1}^{(2)}$ are constants. They are calculated approximately in [6] and depend on the fiber radius R, on the fiber volume content $v = |Y_2|/|Y|$, and also on the dimensionless parameter $\ae = (\varepsilon_{*1} - \varepsilon_{*2})/(\varepsilon_{*1} + \varepsilon_{*2})$ for electrodynamic problem or on $\ae = (\lambda_1 - \lambda_2)/(\lambda_1 + \lambda_2)$ for heat-conduction problems.

Using relations (3.1), one can calculate the specific heat-release intensity $\langle D^{(0)}\rangle$ from (2.4). Upon integration, we obtain that among $\langle\Delta_{ij}\rangle$, only

$$\langle\Delta_{11}\rangle = \langle\Delta_{22}\rangle = \varepsilon''_2 v\Delta, \quad \langle\Delta_{33}\rangle = \varepsilon''_2 v \quad \Big(\Delta = 1 + A_1^{(2)} + \bar{A}_1^{(2)} + \sum_{n=0}^{\infty}(2n+1)A_{2n+1}^{(2)}\bar{A}_{2n+1}^{(2)}\Big)$$

are nonzero. Therefore,

$$\langle D^{(0)}\rangle = (1/2)\,\chi^2\omega\varepsilon_2''v\,(\Delta\langle E_1^{(0)}\rangle^2 + \Delta\langle E_2^{(0)}\rangle^2 + \langle E_3^{(0)}\rangle^2).$$

If the solutions of [6] for regular arrangements of fibers are used, for Δ we have the approximate expression

$$\Delta = 1 + (2k-1)g_k^2 R^{4k} B + O(R^{8k}),$$

where

$$B = \frac{æ^2(1+æ)}{(1+æv)^2} + \frac{\bar{æ}^2(1+\bar{æ})}{(1+\bar{æ}v)^2} + \frac{æ\bar{æ}(1+æ)(1+\bar{æ})}{(1+æv)(1+\bar{æ}v)};$$

$k = 2$ for the square and $k = 3$ for the triangular arrangement of fibers; the values of the constants g_k are presented in [7]; and $æ = (\varepsilon_{*1} - \varepsilon_{*2})/(\varepsilon_{*1} + \varepsilon_{*2})$ (no summation over k).

Using the results of [6] and the above analogy, we write the effective heat-conductivities of the composite $\widehat{\lambda}_{ij}$ as

$$\widehat{\lambda}_{11} = \widehat{\lambda}_{22} = \lambda_1\Big(\frac{1-æv}{1+æv} - A\,\frac{æ^3 v^{2k+1}}{(1+æv)^2}\Big) + O(v^{4k}),\ \ \widehat{\lambda}_{33} = \lambda_1(1-v)+\lambda_2 v,\ \ \widehat{\lambda}_{ij} = 0\ \ (i \neq j).$$

Here $æ = (\lambda_1 - \lambda_2)/(\lambda_1 + \lambda_2)$; $A = 0.612$ and $k = 2$ for the square arrangement of fibers and $A = 0.151$ and $k = 3$ for the triangular arrangement.

4. We estimate the local temperature differences inside a composite unit cell in the macrouniform temperature field (2.19).

We consider a composite with a regular square arrangement of fibers heated by a field with the electric component $\mathbf{E}$ directed across the fibers in the x_1 direction:

$$T = T_0(t) + \frac{1}{2}\alpha^2\chi^2(t)\omega\nu_{11}(\mathbf{y})\langle E_1^{(0)}\rangle^2 + \dots\,. \tag{4.1}$$

To solve the formulated problem, one should construct a function $\nu_{11}(\mathbf{y})$ that is a doubly periodic function in the matrix Y_1 and, following (2.17), satisfies the Poisson equation with the constant right-hand side

$$\lambda_1\,\Delta\nu_{11}^{(1)} = A_1 = \text{const}\quad \Big(A_1 = \frac{\rho_1 c_1}{\langle\rho c\rangle}\langle\Delta_{11}\rangle = \frac{\rho_1 c_1 v\varepsilon_2''\Delta}{\langle\rho c\rangle}\Big). \tag{4.2}$$

In the fiber Y_2, ν_{11} is a regular function that satisfies the Poisson equation with the variable right-hand side

$$\lambda_2\Delta\nu_{11}^{(2)} = A_2 - \varepsilon_2''\varphi_2(r,\theta), \tag{4.3}$$

where $A_2 = (\rho_2 c_2 v\varepsilon_2''\Delta)/\langle\rho c\rangle$;

$$\varphi_2 = 1 + \Phi_{1|1}^{(2)} + \overline{\Phi}_{1|1}^{(2)} + \Phi_{1|1}^{(2)}\overline{\Phi}_{1|1}^{(2)} + \Phi_{1|2}^{(2)}\overline{\Phi}_{1|2}^{(2)}; \tag{4.4}$$

and the function $\Phi_1^{(2)}$ is given by formula (3.1)

At the interface Γ $(r = R)$, conditions (2.22), which in this case reduce to

$$\nu_{11}^{(1)} = \nu_{11}^{(2)},\quad \lambda_2\,\frac{\partial\nu_{11}^{(2)}}{\partial r} = \lambda_2\,\frac{\partial\nu_{11}^{(1)}}{\partial r}, \tag{4.5}$$

should be satisfied. One should supplement (4.2)–(4.5) by the condition

$$\langle\nu_{11}\rangle = 0. \tag{4.6}$$

Substituting representations (3.1) into (4.4), we obtain

$$\varphi_2 = 1 + A_1^{(2)} + \overline{A}_1^{(2)} + A_1^{(2)}\overline{A}_1^{(2)} + 9A_3^{(2)}\overline{A}_3^{(2)}(r/R)^4 + 3(A_3^{(2)} + \overline{A}_3^{(2)} + A_1^{(2)}\overline{A}_3^{(2)}$$
$$+ \overline{A}_1^{(2)}A_3^{(2)})(r/R)^2\cos 2\theta + 7(A_7^{(2)} + \overline{A}_7^{(2)} + A_1^{(2)}\overline{A}_7^{(2)} + \overline{A}_1^{(2)}A_7^{(2)})(r/R)^6\cos 6\theta + \dots\,. \tag{4.7}$$

For the composite with a square arrangement of fibers, with allowance for the coefficients calculated in [6], Eq. (4.7) is written as

$$\varphi_2 = a + bR^8 + dR^8(r/R)^4 - 3Ag_2R^4(r/R)^2\cos 2\theta - 7Ag_4R^8(r/R)^6\cos 6\theta + O(R^{12}) \qquad (4.8)$$

[the coefficients a, b, d, and A are not included in the final expression (4.11) for T, so the formulas for them are not presented herein].

The general doubly periodic solution of the Poisson equation (4.2) is constructed as the sum of the partial doubly periodic solution of the Poisson equation and the arbitrary doubly periodic harmonic function

$$\lambda_1\nu_{11}^{(1)} = \frac{1}{4}A_1\Big(z\bar{z} - \frac{2}{\pi}\,\mathrm{Re}\,\nu(z)\Big) + \sum_{k=0}^{\infty} D_{2k+2}\mathrm{Re}\,\frac{P^{(2k)}(z)}{(2k+1)!}, \qquad (4.9)$$

where $\nu(z) = \int\zeta(z)\,dz$; $\zeta(z)$ and $P(z)$ are the Weierstrass functions. The regular solution of the Poisson equation (4.3) written with allowance for (4.8) is of the form

$$\lambda_2\nu_{11}^{(2)} = \frac{1}{4}\,(A_2 - \varepsilon_2''(a + bR^8))r^2 - \frac{1}{36}\,\varepsilon_2''dR^4r^6 + \frac{1}{4}\,\varepsilon_2''g_2AR^2r^4\cos 2\theta$$
$$+ \frac{1}{4}\,\varepsilon_2''g_4AR^2r^8\cos 6\theta + O(R^{12}) + \sum_{k=0}^{\infty} C_{2k}(r/R)^{2k}\cos 2k\theta. \qquad (4.10)$$

Substituting (4.9) and (4.10) into conditions (4.5) and (4.6) and equating the coefficients of $\cos 2k\theta$ ($k = 0, 1, 2, \ldots$), we obtain an infinite system of linear algebraic equations for the coefficients D_2, $D_4, \ldots, C_0$, C_2, $C_4, \ldots$ that depend on the small parameter R ($0 < R < 1/2$). Analysis of this system shows that with a decrease in R all these coefficients decrease not slower than R^4, except for the coefficient C_0, which increases without bound: $C_0 = [-A_1/(2\pi)](\lambda_2/\lambda_1)\ln R$. If we consider only this higher term in expansion (4.10), then $\nu_{11}^{(2)} = [-A_1/(2\pi\lambda_1)]\ln R + \ldots$. In this case, we find from (4.1) that the fiber temperature is expressed as

$$T = T_0(t) + \frac{\varepsilon_2''\gamma}{4\lambda_1}\,\chi^2(t)\omega\alpha^2R^2\ln\frac{1}{R}\langle E_1^{(0)}\rangle^2 + \ldots\,. \qquad (4.11)$$

Here

$$\gamma = \rho_1c_1/\langle\rho c\rangle = \rho_1c_1/(\rho_1c_1(1 - \pi R^2) + \rho_2c_2\pi R^2).$$

Analysis of (4.9) shows that the temperature distribution in the matrix is of the form

$$T = T_0(t) + \frac{\varepsilon_2''\gamma}{4\lambda_1}\,\chi^2(t)\omega\alpha^2R^2\ln\frac{1}{r}\langle E_1^{(0)}\rangle^2 + \ldots\,.$$

5. By way of example, we consider a polymer carbon-fiber composite with epoxy resin as a binder. The volume content of the fiber in the composite is $v = 0.6$. The fiber parameters are as follows: density $\rho_2 = 1700$ kg $\cdot$ m^{-3}, heat capacity $c_2 = 1.07$ kJ/(kg $\cdot$ K), radius $\alpha R = 1.3\cdot 10^{-6}$ m, and dielectric-loss factor $\varepsilon_2'' = 4.1\cdot 10^{-5}$ S $\cdot$ sec/m. The characteristics of the epoxy resin are: density $\rho_1 = 2168$ kg $\cdot$ m^{-3}, heat capacity $c_1 = 0.5$ kJ/(kg$\cdot$K), and heat conductivity $\lambda_1 = 0.21$ W/(m$\cdot$K). The composite is heated in an electromagnetic field with average electric-component intensity $E = 100$ V $\cdot$ m^{-1} and frequency $\omega = 2.45\cdot 10^9$ sec^{-1}. The amplitude of the field remains unchanged with time.

Substitution of the indicated values in (4.11) yields a fiber-temperature nonuniformity $\Delta T = 1.31\cdot 10^{-5}$ K. This value is vanishingly small compared with the average temperature $T_0(t)$, which should be 50–70°C according to the heat-treatment technology.

An increase in the fiber radius to $\alpha R = 0.5\cdot 10^{-3}$ m increases the temperature nonuniformity of the periodicity cell to $\Delta T = 1.9$ K. This nonuniformity is noticeable, but is still acceptable technologically, since it is 0.3% of the lower limit of the admissible temperature range. Under these conditions, an increase in the field intensity, for example, by a factor of 2 (which corresponds to an increase in the supplied SHF energy by a factor of 4 and to an equivalent increase in the heating rate) increases the nonuniformity to $\Delta T = 7.6$ K. This is 10% higher than the upper limit of the admissible range and is technologically on the verge of permissible values, i.e., a further increase in the supplied intensity is inadmissible.

The superheating values were verified experimentally as follows. A unidirectional carbon-fiber composite on unsolidified epoxy resin (with the above characteristics) with a fiber radius of $0.5 \cdot 10^{-3}$ m and an overall thickness $(2\text{–}3) \cdot 10^{-3}$ m in the form of a 0.2×0.1 m plate was placed at the opening of the SHF-source horn. The field intensity was measured by a special gauge. Uniform drying and polymerization over the entire plate thickness were observed under irradiation by a field with intensity of from 50 to 200 $V \cdot m^{-1}$. When the intensity exceeds 200 $V \cdot m^{-1}$, the composite plate is buckled, and local burns-through appear in its thickness.

These results indicate that the proposed mathematical model is applicable for predicting temperature fields under SHF heating of composites and for determining admissible SHF regimes.

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